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Localized Massive Gravity from B fields

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ABSTRACT

We consider five-dimensional domain-wall solutions which arise from a sphere reduction in M-theory or string theory and have the higher-dimensional interpretation of the near-horizon region of various p-branes in constant, background B fields. We analyze the fluctuation spectrum of linearized gravity and find that there is a massive state which is localized and plays the role of the four-dimensional graviton.

1 Introduction

Randall and Sundrum [1, 2] have shown that, with fine tuned brane tension, a flat 3-brane embedded in AdS_5 can have a single, massless bound state. Four-dimensional gravity is recovered at low-energy scales. It has also been proposed that part or all of gravitational interactions are the result of massive gravitons. For example, in one model, gravitational interactions are due to the net effect of the massless graviton and ultra-light Kaluza-Klein state(s) [3, 4, 5, 6]. In another proposal, there is no normalizable massless graviton and four-dimensional gravity is reproduced at intermediate scales from a resonance-like behavior of the Kaluza-Klein wave functions [5, 6, 7, 8, 9].

Recently, it has been shown that an AdS_4 brane in AdS_5 does not have a normalizable massless graviton. Instead, there is a very light, but massive bound state graviton mode, which reproduces four-dimensional gravity [10, 11, 12, 13]. The bound state mass as a function of brane tension, as well as the modified law of gravity, were explored in [14, 15].

A brane world in which two extra infinite spatial dimensions do not commute has recently been used in order to localize scalar fields as well as fermionic and gauge fields [19]. Also, the cosmological evolution of the four-dimensional universe on the probe D3-brane in geodesic motion in the curved background of the source Dp -brane with nonzero NS B field has been explored [20].

In this paper, we consider brane world models which arise from a sphere reduction in M-theory or string theory, as the near-horizon of p -branes [16] with a constant, background B -field on the world-volume. The dual field theory is non-commutative Yang-Mills [17, 18].

We consider string theoretic p -brane solutions for $p = 3, 4, 5$ with 0, 1, 2 world-volume dimensions wrapped around a compact manifold, respectively. For $p > 5$, the space of the extra large dimension has finite-volume, with a naked singularity at the end, for which case the localization of gravity is trivial.

For all of the cases we study, we find that there is no normalizable massless graviton, but there is a massive bound state graviton which plays the role of the four-dimensional graviton. This yields another class of examples for which gravitational interactions are entirely the result of massive gravitons. As we will see, the bound mass increases continuously from zero as a constant, background B or E -field is turned on, except for the case $p = 5$, for which there is a mass gap.

This paper is organized as follows. In section 2, we find the graviton wave equation in the background of five-dimensional domain-walls that originate in string theory as p -branes in constant background B and E -fields. In section 3, we show that the massless graviton

wave function is not normalizable for nonzero background B or E fields, indicating that the massless graviton is not localized on the brane. In section 4, we show that there is a massive graviton localized on the brane for nonzero B and E -fields. Lastly, an appendix shows how the graviton wave equation can be solved exactly for $p = 5$.

2 The graviton wave equation

2.1 With background B field

Consider the metric of a p -brane (expressed in the string frame) in a constant NS B field in the 2,3 directions [18, 21]:

$$\begin{aligned} ds_{10}^2 &= H^{-1/2}(-dt^2 + dx_1^2 + h(dx_2^2 + dx_3^2) + dy_i^2) + l_s^4 H^{1/2}(du^2 + u^2 d\Omega_5^2), \\ H &= 1 + \frac{R^{7-p}}{l_s^4 u^{7-p}}, \quad h^{-1} = H^{-1} \sin^2 \theta + \cos^2 \theta, \end{aligned} \quad (2.1)$$

where $u = r/l_s^2$ is a dimensionless radial parameter and $i = 1, \dots, p-3$. The B field and dilaton are given by

$$B_{23} = \tan \theta H^{-1} h, \quad e^{2\varphi} = g^2 H^{(3-p)/2} h, \quad (2.2)$$

$$\cos \theta R^{7-p} = (4\pi)^{(5-p)/2} \Gamma\left(\frac{7-p}{2}\right) g_s l_s^{p-3} N, \quad (2.3)$$

where N is the number of p -branes and g_s is the asymptotic value of the coupling constant. θ is known as the non-commutativity parameter, and is related to the asymptotic value of the B -field: $B_{23}^\infty = \tan \theta$.

Consider the following decoupling limit, in which we take the B field to infinity [18]:

$$\begin{aligned} l_s &\rightarrow 0, \quad l_s^2 \tan \theta = b, \\ \tilde{x}_{2,3} &= \frac{b}{l_s^2} x_{2,3}, \end{aligned} \quad (2.4)$$

where b, u, \tilde{x}_μ and $g_s l_s^{p-5}$ stay fixed. Thus,

$$l_s^{-2} ds^2 = \left(\frac{u}{R}\right)^{(7-p)/2} (-dt^2 + dx_1^2 + \tilde{h}(d\tilde{x}_2^2 + d\tilde{x}_3^2) + dy_i^2) + \left(\frac{R}{u}\right)^{(7-p)/2} (du^2 + u^2 d\Omega_{8-p}^2), \quad (2.5)$$

where $\tilde{h}^{-1} = 1 + b^2(u/R)^{7-p}$. The dual field theory is Yang-Mills with noncommuting 2,3 coordinates [18]. For small u , the above solution reduces to $AdS_5 \times S^5$, corresponding to ordinary Yang-Mills living in the IR region of the dual field theory, which is also the case if we had taken the decoupling limit with finite B -field. From this point on, we drop the \sim on the coordinates. For $p \neq 5$, we change coordinates to $u/R \equiv (1 + k|z|)^{2/(p-5)}$ [16]:

$$l_s^{-2} ds^2 = (1 + k|z|)^{\frac{7-p}{p-5}} (-dt^2 + dx_1^2 + \tilde{h}(dx_2^2 + dx_3^2) + dy_i^2 + dz^2) + (1 + k|z|)^{\frac{p-3}{p-5}} d\Omega_{8-p}^2, \quad (2.6)$$

For $p = 5$, we change coordinates to $u/R \equiv e^{-k|z|/2}$ [16]:

$$l_s^{-2} ds^2 = e^{-k|z|/2} (-dt^2 + dx_1^2 + \tilde{h}(dx_2^2 + dx_3^2) + dy_i^2 + dz^2 + d\Omega_3^2). \quad (2.7)$$

After dimensional-reduction over S^{8-p} , the above metric corresponds to a $p+2$ -dimensional domain wall at $z = 0$. We will consider the case where y_i are wrapped around a compact manifold, so that there is a $1+3$ -dimensional dual field theory on the remaining world-volume coordinates. The equation of motion for the graviton fluctuation $\Phi = g^{00} h_{01}$ is

$$\partial_M \sqrt{-g} e^{-2\varphi} g^{MN} \partial_N \Phi = 0, \quad (2.8)$$

where h_{01} is associated with the energy-momentum tensor component T_{01} of the Yang-Mills theory. For $p \neq 5$, consider the ansatz

$$\Phi = \phi(z) e^{ip \cdot x} = (1 + k|z|)^{\frac{9-p}{2(5-p)}} \psi(z) e^{ip \cdot x}. \quad (2.9)$$

Thus,

$$-\psi'' + U\psi = m^2 \psi, \quad (2.10)$$

$$U = \frac{(p-9)(3p-19)k^2}{4(5-p)^2(1+k|z|)^2} + \frac{p-9}{5-p} k\delta(z) + \frac{\alpha^2}{(1+k|z|)^{2(7-p)}5-p}, \quad (2.11)$$

where

$$\alpha \equiv b\sqrt{(p_2^2 + p_3^2)}, \quad (2.12)$$

and $m^2 = -p^2$.

For $p = 5$, consider the ansatz

$$\Phi = \phi(z) e^{ip \cdot x} = e^{k|z|/2} \psi(z) e^{ip \cdot x}. \quad (2.13)$$

Thus, ψ satisfies (2.10) with

$$U = \frac{1}{4}k^2 - k\delta(z) + \alpha^2 e^{-k|z|}. \quad (2.14)$$

For more general directions of the world-volume B field, $\alpha \equiv b\sqrt{p_i^2}$, where p_i are the momenta along the large, non-commuting directions in the world-volume. Thus, if only the wrapped y_i coordinates are non-commuting, then $\alpha = 0$.

2.2 With background B and E fields

Consider the metric of a p -brane in a constant two-component NS B field background [18, 21]:

$$\begin{aligned} ds^2 &= H^{-1/2} (h_e(-dt^2 + dx_1^2) + h_m(dx_2^2 + dx_3^2) + dy_i^2) + l_s^4 H^{1/2} (du^2 + u^2 d\Omega_{8-p}^2), \\ H &= 1 + \frac{R^{7-p}}{l_s^4 u^{7-p}}, \quad h_m^{-1} = H^{-1} \sin^2 \theta_m + \cos^2 \theta_m, \quad h_e^{-1} = -H^{-1} \sinh^2 \theta_e + \cosh^2 \theta_e. \end{aligned} \quad (2.15)$$

The B and E field components and dilaton are given by

$$B_{23} = \tan \theta_m H^{-1} h_m, \quad E_{01} = -\tanh \theta_e H^{-1} h_e, \quad e^{2\varphi} = g^2 H^{\frac{3-p}{2}} h_e h_m. \quad (2.16)$$

Consider the following decoupling limit [18]:

$$l_s \rightarrow 0, \quad \cosh \theta_e = \frac{b'}{l_s}, \quad \cos \theta_m = \text{fixed}, \quad (2.17)$$

$$x_{0,1} = \frac{b'}{l_s^2} \tilde{x}_{0,1}, \quad x_{2,3} = l_s \cos \theta_m \tilde{x}_{2,3},$$

where b', u, \tilde{x}_μ and $g_s l_s^{p-7}$ remain fixed. Thus,

$$l_s^{-2} ds^2 = H^{1/2} \left(\left(\frac{u}{R} \right)^{7-p} (-dt^2 + d\tilde{x}_1^2) + \left(\frac{u}{R} \right)^{7-p} \tilde{h}_m (d\tilde{x}_2^2 + d\tilde{x}_3^2) + H^{-1} dy_i^2 + du^2 + u^2 d\Omega_{8-p}^2 \right), \quad (2.18)$$

where we define

$$\tilde{h}_m^{-1} = 1 + \left(\frac{u}{R} \right)^{7-p} \cos^{-2} \theta_m. \quad (2.19)$$

The dual field theory is Yang-Mills with noncommuting $0, 1$ and $2, 3$ coordinates [18]. Once again, we drop the \sim on the coordinates. For $p \neq 5$, we change coordinates to $u/R \equiv (1 + k|z|)^{2/(p-5)}$:

$$l_s^{-2} ds^2 = H^{1/2} \left((1+k|z|)^{\frac{2(p-7)}{5-p}} (-dt^2 + dx_1^2 + \tilde{h}_m (dx_2^2 + dx_3^2) + dz^2) + H^{-1} dy_i^2 + (1+k|z|)^{\frac{4}{p-5}} d\Omega_{8-p}^2 \right). \quad (2.20)$$

For $p = 5$, we change coordinates to $u/R \equiv e^{-k|z|/2}$:

$$l_s^{-2} ds^2 = H^{1/2} (e^{-k|z|} (-dt^2 + dx_1^2 + \tilde{h}_m (dx_2^2 + dx_3^2) + dz^2 + d\Omega_3^2) + H^{-1} dy_i^2). \quad (2.21)$$

As in the case of a one-component B -field, we consider a dimensional-reduction over S^{8-p} and y_i wrapped around a compact manifold. One effect of the E -field, as opposed to the B -field, is to add a breathing-mode to S^{8-p} . For $p \neq 5$, we insert this metric into the graviton equation of motion (2.8) with the ansatz (2.9) and obtain (2.10) with U given by (2.11) and with

$$\alpha \equiv \frac{\sqrt{p_2^2 + p_3^2}}{\cos \theta_m}. \quad (2.22)$$

For $p = 5$, we use the wave function ansatz (2.13) and obtain (2.10) with U given by (2.14). Again, in general $\alpha \equiv \sqrt{p_i^2} / \cos \theta_m$, where p_i are the momenta along the large, non-commuting directions.

In this paper, we will focus on the $p = 3$ case, for which the volcano potential U is plotted in Figure 1, for $\alpha = 0$ (solid line) and $\alpha = 2$ (dotted line). As can be seen, the effect of the B and E fields is to raise the peak of the potential. This is the case in general for $p \leq 5$.

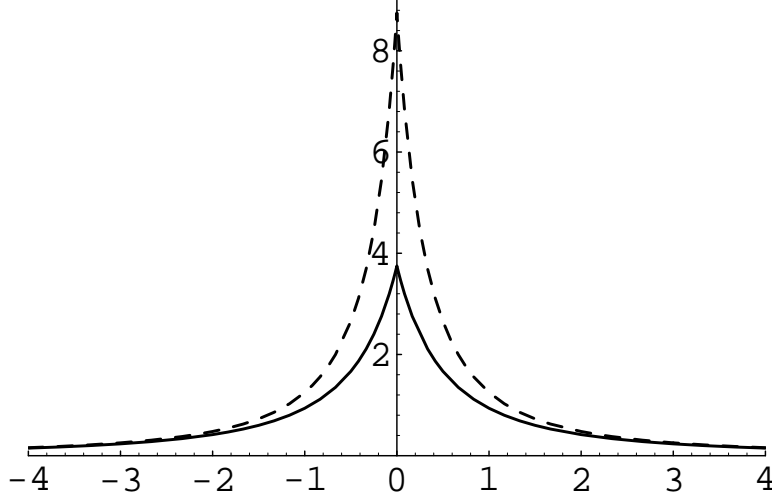


Figure 1: $U(z)$ for $\alpha = 0$ (solid line) and $\alpha = 2$ (dotted line) for $p = 3$.

3 The massless graviton

Consider the coordinate transformation

$$u/R \equiv (1 + k|w|)^{-1} \quad (3.1)$$

and the wave function transformation

$$\Theta(w) = (1 + k|w|)^{\frac{p-3}{4}} \psi(z). \quad (3.2)$$

applied to the graviton wave equation (2.10). Thus,

$$-\Theta'' + V\Theta = \frac{m^2}{(1 + k|w|)^{p-3}} \Theta, \quad (3.3)$$

where

$$V = \frac{(6-p)(8-p)k^2}{4(1 + k|w|)^2} - (6-p)k\delta(w) + \frac{\alpha^2}{(1 + k|w|)^4}. \quad (3.4)$$

For the massless case, this is a Schrödinger-type equation. The wave function solution can be written in terms of Bessel functions as

$$\Theta(w) = N_0(1 + k|w|)^{1/2} [J_\nu\left(\frac{i\alpha}{1 + k|w|}\right) + A(\alpha)Y_\nu\left(\frac{i\alpha}{1 + k|w|}\right)], \quad (3.5)$$

where $\nu = (7-p)/2$ and N_0 is the normalization constant, in the event that the above solution is normalizable. $A(\alpha)$ is solved by considering the $\delta(w)$ boundary condition:

$$A(\alpha) = \frac{2\nu J_\nu(i\alpha) + i\alpha[J_{\nu+1}(i\alpha) - J_{\nu-1}(i\alpha)]}{-2\nu Y_\nu(i\alpha) + i\alpha[Y_{\nu-1}(i\alpha) - Y_{\nu+1}(i\alpha)]}, \quad (3.6)$$

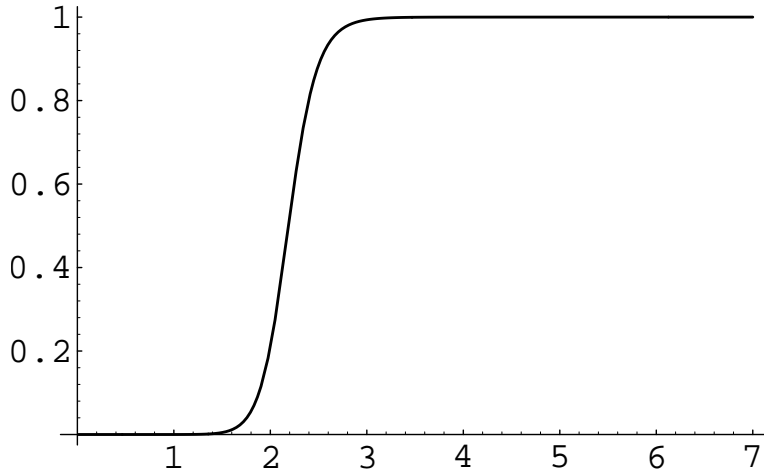


Figure 2: $|A|^2$ versus α for $p = 3$.

where we used $2J'_n(x) = J_{n-1}(x) - J_{n+1}(x)$, and likewise for $Y'_n(x)$. In order for the massless graviton to be localized on the brane, the corresponding wave function must be normalizable, i.e., $\int_0^\infty |\psi(w, m=0)|^2 dw < \infty$. (3.5) can be written in the form

$$\Theta(w) = (\text{normalizable part}) + A(\text{non-normalizable part}), \quad (3.7)$$

as can be seen from the asymptotic forms of the Bessel functions. Thus, the massless graviton is only localized on the brane if $A = 0$. In this limit, there are no background B or E fields and we recover the Randall-Sundrum model.

As shown in Figure 2, for the case of the D3-brane, as α increases, the modulus of A approaches unity. A has the same characteristics for $p \leq 5$ in general. The issue arises as to whether there is a massive graviton state bound to the brane for nonzero A .

4 Localization of the massive graviton

We will first concentrate on the case of the brane world as the dimensionally-reduced near-horizon of the D3-brane. For this case, the massive wave equation (2.10) with U given by (2.11) is a modified Mathieu's equation [22], supplemented by the $\delta(w)$ boundary condition. The exact solution of the modified Mathieu's equation is known, and has been applied to absorption by $D3$ -branes and six-dimensional dyonic strings [23, 24, 25, 26, 27]. However, it is rather laborious to use the exact solution in this context and so we content ourselves with plotting numerical solutions—except for the case $p = 5$, for which the exact solution is a sum of Bessel functions (see appendix A).

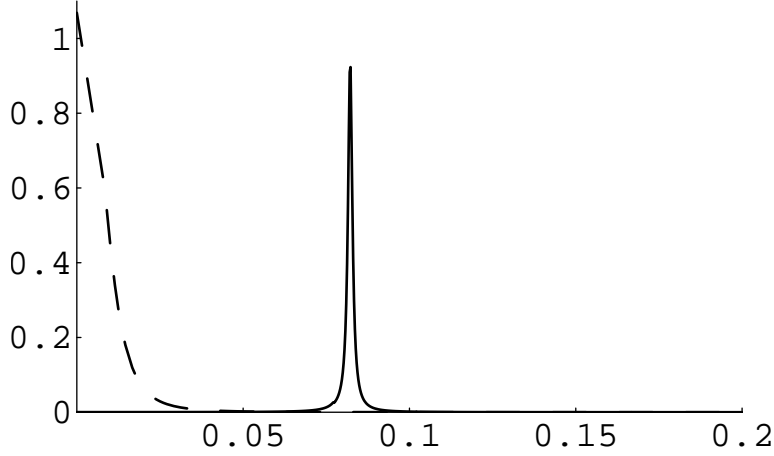


Figure 3: $|\psi(z=0)|^2$ versus m for $\alpha = 0$ (dotted line) and $\alpha = .02$ (solid line) for $p = 3$.

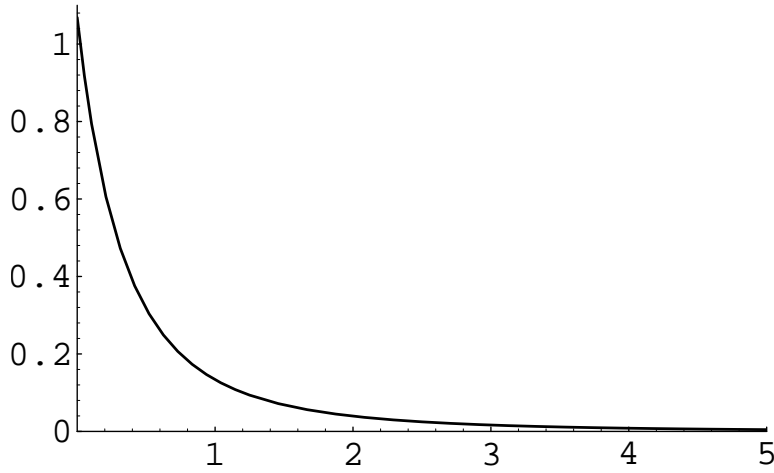


Figure 4: $|\psi(z)|^2$ for $m/k = 0$ for $p = 3$.

In order to solve for the unnormalized wave function, we input the $\delta(z)$ boundary condition and solve the Schrödinger-type equation outwards. We then numerically integrate in order to find the correct normalization factor.

As can be seen from the dotted line in Figure 3, for the case of the D3-brane with no background B and E fields, there is a resonance in the modulus of the wave function on the brane for $m = 0$, which implies that the massless graviton is localized on the brane. This behavior can also be seen from Figure 2, by the fact that $A = 0$ and the massless solution is normalizable.

We may arrive at the same conclusion via Figures 4 and 5, where the massless wavefunction has a peak on the brane at $z = 0$ whereas the wavefunction of mass $m = 1.5k$ oscillates without feeling the presence of the brane [10].

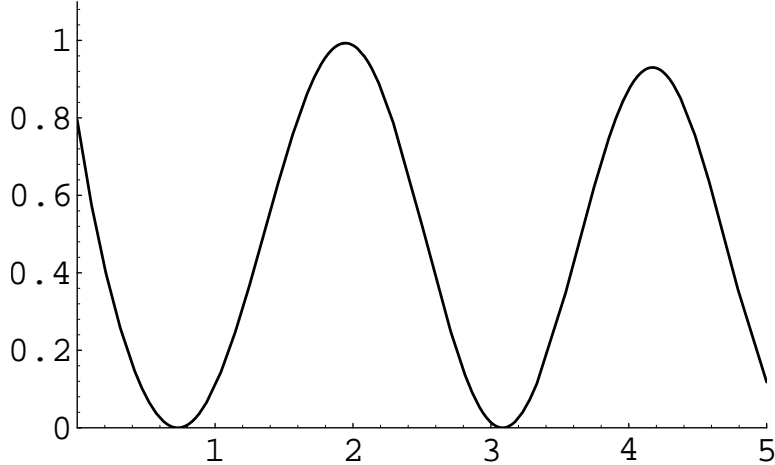


Figure 5: $|\psi(z)|^2$ for $m/k = 1.5$ for $p = 3$.

In the case of nonzero B or E fields, the resonance in the modulus of the wave function on the brane is at a nonzero mass. Thus, for a nonzero, constant background B or E field, a massive graviton is localized on the brane. This behavior is shown by the solid line in Figure 3, where the $|\psi(z = 0)|^2$ is plotted for $\alpha = .02$, and there is a large resonance in the wave function on the brane for $m/k = .0822$. Again, this behavior may also be ascertained from plots of $|\psi(z)|^2$ for various values of m/k . For a mass significantly less than the resonance mass, the wave function has a peak at $z = 0$ that is not relatively high, while a wave function with a mass significantly greater than the resonance mass oscillates without feeling the brane.

It is straight-forward to repeat the above analysis for $p = 4, 5$. Note that a D4-brane reduced on $S^1 \times S^4$ yields the same graviton wave equation as a M5-brane reduced on $T^2 \times S^4$. Likewise, the D5-brane reduced on $T^4 \times S^3$ and a M5-brane intersection reduced on $T^4 \times S^2$ or $K3 \times S^2$ yield the same graviton wave equation. This was initially observed in [16] in the case of zero B -field. Figure 6 shows the resonance mass versus α for $p = 3, 4, 5$. We classify them by the corresponding near-horizon brane configurations in ten or eleven dimensions. The solid line corresponds to the D3-brane. The dotted line corresponds to the D4 or M5-brane. The bold line corresponds to the D5-brane or an M5-brane intersection. It is rather curious that the three lines appear to intersect at the same point, at approximately $\alpha = 2.688$ and $m/k = .976$.

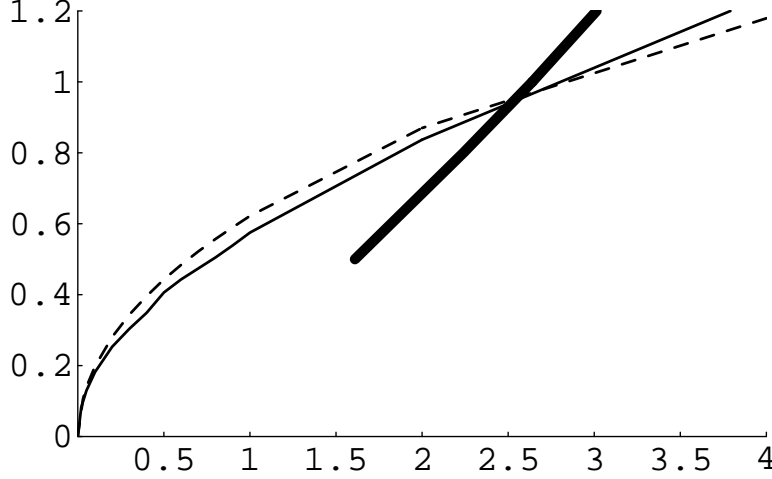


Figure 6: Resonance mass versus α for $p = 3$ (solid line), 4 (dotted line) and 5 (bold line).

A $p = 5$ as an exactly solvable model

Consider the equations (3.3) and (3.4). Although this is not in Schrödinger form, for $p = 5$ this equation is easily solvable for the massive case. The solution for Θ is

$$\Theta = N(1 + k|w|)^{1/2} [J_{-i\gamma} \left(\frac{i\alpha}{1 + k|w|} \right) + A(\alpha) Y_{-i\gamma} \left(\frac{i\alpha}{1 + k|w|} \right)], \quad (\text{A.1})$$

where

$$A(\alpha) = \frac{2J_{-i\gamma}(i\alpha) + i\alpha[J_{-i\gamma+1}(i\alpha) - J_{-i\gamma-1}(i\alpha)]}{-2Y_{-i\gamma}(i\alpha) + i\alpha[Y_{-i\gamma-1}(i\alpha) - Y_{-i\gamma+1}(i\alpha)]}, \quad (\text{A.2})$$

and $\gamma \equiv \sqrt{4(m/k)^2 - 1}$. Transforming back to the z coordinate and ψ wave function using (3.1) and (3.2) we obtain the wave function solution

$$\psi(z) = N_0 [J_{-i\gamma}(i\alpha e^{-k|z|/2}) + A(\alpha) Y_{-i\gamma}(i\alpha e^{-k|z|/2})]. \quad (\text{A.3})$$

For the massless graviton

$$\psi(z) \sim e^{-k|z|/2} + A(\alpha) e^{k|z|/2}, \quad (\text{A.4})$$

and so the massless graviton wave function is not normalizable for nonzero B or E fields.

As in the zero B -field case, there is a mass gap and so we must have $m^2 \geq \frac{1}{4}k^2$ for the massive wave functions. Expanding the Bessel functions for large $k|z|$ yields

$$\begin{aligned} \psi(z) = & N \frac{e^{\pi/2}}{\Gamma(1 - i\gamma)} \left([1 + iA(\alpha) \text{csch}(\pi\gamma) \cosh(\pi\gamma)] e^{iq|z| - i\gamma \ln(\alpha/2)} - \right. \\ & \left. iA(\alpha) \text{csch}(\pi\gamma) e^{-\pi} \frac{\Gamma(1 - i\gamma)}{\Gamma(1 + i\gamma)} e^{-iq|z| + i\gamma \ln(\alpha/2)} \right), \end{aligned} \quad (\text{A.5})$$

where $q \equiv \sqrt{m^2 - k^2/4}$. We numerically normalize the wave function, and find the resonant mass by solving for the maximum of $|\psi(z=0)|^2$ for a given α . The resonant mass versus

α is plotted for $p = 5$ as the bold line in Figure 6. With this exact solution to the wave function, one can then find an analytic expression for how the gravitational potential is modified by the presence of the constant, background B and E fields.

Note that, for the case of zero B -field, the $p = 5$ wave equation that results from not dropping the "1" in the harmonic function H is also exactly solvable as a sum of Bessel functions [28], using the w coordinate. This is of interest if one wishes to explore the localization of gravity analytically in two-scalar domain-wall solutions [16].

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References

- [1] L. Randall and R. Sundrum, *An alternative to compactification*, hep-th/9906064.
- [2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, Phys. Rev. Lett. 83 (1999) 3370, hep-ph/9905221.
- [3] I.I. Kogan, S. Mouslopoulos, A. Papazoglo, G.G. Ross and J. Santiago, *A three three-brane universe: New phenomenology for the new millenium?*, Nucl. Phys. B584 (2000) 313, hep-th/9912552.
- [4] S. Mouslopoulos and A. Papazoglou, *"++" brane model phenomenology*, JHEP 0011 (2000) 018, hep-th/0003207.
- [5] I.I. Kogan and G.G. Ross, *Brane universe and multigravity: Modification of gravity at large and small distances*, Phys. Lett. B485 (2000) 255, hep-th/0003074.
- [6] I.I. Kogan, S. Mouslopoulos, A. Papazoglou and G.G. Ross, *Multi-brane worlds and modification of gravity at large scales*, hep-th/0006030.
- [7] R. Gregory, V.A. Rubakov and S.M. Sibiryakov, *Opening up extra dimensions at ultra-large scales*, Phys. Rev. Lett. 84 (2000) 5928, hep-th/0002072.
- [8] C. Csáki, J. Erlich and T.J. Hollowood, *Quasi-localization of gravity by resonant modes*, Phys. Rev. Lett. 84 (2000) 5932, hep-th/0002161.

- [9] G. Dvali, G. Gabadadze and M. Porrati, *Metastable gravitons and infinite volume extra dimensions*, Phys. Lett. B484 (2000) 112, hep-th/0002190.
- [10] A. Karch and L. Randall, *Locally localized gravity*, hep-th/0011156.
- [11] I.I. Kogan, S. Mouslopoulos and A. Papazoglou, *The $m \rightarrow 0$ limit for massive in dS_4 and AdS_4 : How to circumvent the van Dam-Veltman-Zakharov discontinuity*, to be published in Phys. Lett. B, hep-th/0011138.
- [12] I.I. Kogan, S. Mouslopoulos and A. Papazoglou, *A new bigravity model with exclusively positive branes*, to be published in Phys. Lett. B, hep-th/0011141.
- [13] M. Porrati, *No Van Dam-Veltman-Zakharov discontinuity in AdS space*, to be published in Phys. Lett. B, hep-th/0011152.
- [14] M. D. Schwartz, *The emergence of localized gravity*, hep-th/0011177.
- [15] A. Miemiec, *A power law for the lowest eigenvalue in localized massive gravity*, hep-th/0011160.
- [16] M. Cvetič, H. Lü and C.N. Pope, *Domain walls with localised gravity and domain-wall/QFT correspondence*, hep-th/0007209.
- [17] A. Hashimoto and N. Itzhaki, *Non-commutative Yang-Mills and the AdS/CFT Correspondence*, Phys. Lett. B465 (1999) 142-147, hep-th/9907166.
- [18] J.M. Maldacena and J.G. Russo, *Large N limit of non-commutative gauge theories*, hep-th/9908134.
- [19] L. Pilo and A. Riotto, *The non-commutative brane world*, hep-th/0012174.
- [20] D. Youm, *Brane inflation in the background of D-brane with NS B field*, hep-th/0011024.
- [21] M. Alishahiha, Y. Oz and M.M. Sheikh-Jabbari, *Supergravity and large N noncommutative field theories*, JHEP 9911 (1999) 007, hep-th/9909215.
- [22] D.J. Gross, A. Hashimoto and N. Itzhaki, *Observables of non-commutative gauge theories*, hep-th/0008075.
- [23] S.S. Gubser and A. Hashimoto, *Exact absorption probabilities for the D3-brane*, hep-th/9805140.

- [24] M. Cvetič, H. Lü, C.N. Pope and T.A. Tran, *Exact absorption probability in the extremal six-dimensional dyonic string background*, Phys. Rev. D59 (1999) 126002, hep-th/9901002.
- [25] M. Cvetič, H. Lü and J.F. Vázquez-Poritz, *Massive-scalar absorption by extremal p-branes*, Phys. Lett. B462 (1999) 62-69, hep-th/9904135.
- [26] M. Cvetič, H. Lü and J.F. Vázquez-Poritz, *Absorption by extremal D3-branes*, hep-th/0002128.
- [27] D.K. Park, S.N. Tamaryan, H.J.W. Müller-Kirsten and Jian-zu Zhang, *D-branes and their absorptivity in Born-Infeld theory*, Nucl. Phys. B594 (2001) 243-271, hep-th/0005165.
- [28] J.F. Vázquez-Poritz, unpublished notes, 2001.